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Application of ethnomathematics in the teaching of integral calculus at the state polytechnic University of Carchi

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Abstract— The students of the Basic Sciences Center of the State Polytechnic University of Carchi present difficulties in abstract topics in Integral Calculus on volumes of revolution for which the implementation of ethnomathematics is proposed as an alternative to improve learning results. This current seeks to integrate and value the mathematical practices of diverse cultures in teaching, enriching the educational process with a contextualized and diverse approach. How ethnomathematics is intertwined with creativity and culture in clay pot design is explored, highlighting the importance of mathematics in a specific cultural context. The relationship between the Pasto culture and UPEC exemplifies how institutions can collaborate with indigenous communities to preserve their cultural heritage. In addition, integral calculus and its application in vessel design are analyzed, demonstrating that ethnomathematical teaching generates a deeper and more applicable understanding of mathematics, promoting a broad and diverse vision of this discipline and increasing student motivation.

Keywords: Ethnomathematics, Integral Calculus, Teaching, Culture Pasto

INTRODUCTION

Students from the Basic Sciences Center at the State Polytechnic University of Carchi (UPEC), under an agreement with ACOFI (Colombian Association of Engineering Faculties), took the EXIM exam and obtained results similar to the institutional average in Mathematics and Physics. However, the results revealed that students at UPEC had difficulties with topics in Integral Calculus. Therefore, professors proposed the implementation of ethnomathematics as a methodological alternative to improve learning outcomes.

Ethnomathematics is a branch of mathematics education that aims to integrate and value mathematical practices from different cultures within the teaching and learning process. It offers a more contextualized, meaningful, and diverse approach to teaching mathematics. This text explores the foundations and benefits of ethnomathematics as a teaching alternative and analyzes how this approach can positively impact students' understanding, motivation, and appreciation of mathematics.

Clay pot design within the Pasto culture represents a fascinating example of how ethnomathematics intersects with creativity, culture, and everyday mathematics in a community's life. These pots, historically integral to cultural practices, are used to explore the mathematical relevance in a specific cultural context, emphasizing the importance of this perspective in math education.

The relationship between the Pasto culture and UPEC is a clear example of how educational institutions can collaborate with indigenous communities to preserve and promote their cultural heritage. This article highlights their joint initiatives, academic programs, and research efforts that have impacted the region's educational and cultural development.

Integral Calculus is a fundamental branch of mathematics used to solve problems related to area, volume, and rates of change. Among its key techniques is calculating volumes of revolution, applicable when a 2D region is rotated around an axis to form a 3D solid. This concept is used in the context of clay pot design.

Thus, this work aims to promote the contextualization of Pasto culture through ethnomathematics among UPEC students. It also seeks to relate volume of revolution calculations with the design of Pasto clay pots and evaluate how ethnomathematics influences the teaching of this concept in the Integral Calculus course UPEC.

MATERIALS AND METHODS

a. Foundations of Ethnomathematics

Ethnomathematics is based on the idea that mathematics is not merely a set of abstract and universal concepts, but is influenced by the cultural, historical, and social practices of the communities in which it develops. This approach recognizes and values the diversity of ways in which mathematics is expressed in different cultural contexts and seeks to integrate this diversity into the educational process.

One of the pillars of ethnomathematics is the recognition of mathematical knowledge present in non-Western cultures and in the everyday practices of communities. This involves exploring how people use mathematics in their daily activities, traditions, measurement systems, ways of counting and calculating, among other aspects.

b. Benefits of Ethnomathematics in Teaching

1. Relevance and Meaning: By integrating mathematical practices from different cultures into the classroom, ethnomathematics makes mathematics more relevant and meaningful for students. They can relate mathematical concepts to their own reality and experiences, which increases their motivation and understanding.

2. Diversity and Multiculturalism: Ethnomathematics promotes a broader and more diverse view of mathematics by recognizing that there are multiple ways of doing mathematics around the world. This fosters respect for cultural differences and appreciation for the mathematical contributions of various communities.

3. Interdisciplinary Connections: By exploring mathematical practices in different cultural contexts, ethnomathematics also facilitates the integration of mathematics with other disciplines such as history, anthropology, sociology, and geography. This enriches the educational experience by showing the interconnectedness of knowledge and understanding.

4. Development of Critical Thinking: Ethnomathematics encourages students to question and reflect on the ways in which mathematics is taught and applied. This stimulates critical thinking and analytical skills, as students must consider different perspectives and contexts to solve mathematical problems.

5. Inclusion and Equity: By recognizing and valuing the mathematical practices of diverse cultures, ethnomathematics promotes inclusion and equity in the classroom. All students can see their own experiences reflected in the learning process, contributing to a more inclusive and respectful educational environment.

c. Implementation of Ethnomathematics in the Classroom

The implementation of ethnomathematics in the classroom requires a conscious and reflective pedagogical approach. Some effective strategies and practices may include:

1. Exploration of Cultural Practices: Invite students to investigate and share how mathematics is used in their own cultures and communities. This may include activities such as counting in different languages, exploring traditional measurement systems, analyzing patterns in cultural artifacts, among others.
2. Analysis of Contextualized Problems: Present mathematical problems that are related to diverse cultural situations and contexts. For example, solving proportion problems using traditional cooking recipes or calculating areas and perimeters in architectural designs from different cultures.
3. Discussion and Reflection: Encourage classroom discussions on how mathematics is applied and perceived in different cultures. This can include debates on the history of mathematics, the influence of culture on number systems, and the importance of mathematics in solving everyday problems, among other topics.
4. Use of Diverse Resources: Incorporate educational materials and resources that reflect cultural diversity and the multiple ways in which mathematics is expressed. This may include books, videos, games, traditional mathematical artifacts, and other resources.
5. Respect and Appreciation: Promote an environment of respect and appreciation for the various ways of doing mathematics. This involves recognizing and celebrating the mathematical contributions of different cultures, as well as avoiding stereotypes and prejudices regarding the mathematical practices of communities. (Oliveras, 2016)

d. Relationship of Ethnomathematics in the Design of Clay Vessels within the Pasto Culture

Cultural Context of the Pasto Culture

Before delving into the design of clay vessels, it is essential to understand the cultural context of the Pasto culture. This ethnic group is located in the southwest of Colombia, in the Nariño region, and in parts of Ecuador, specifically in the Carchi Province. They are known for their rich cultural heritage, which includes unique traditions, art, music, and crafts.

Within this culture, the art of ceramics has been an ancestral and meaningful practice. The clay vessels crafted by Pasto artisans not only serve a utilitarian function but also represent a form of cultural expression, incorporating symbols, patterns, and techniques passed down from generation to generation.

- i. **Ethnomathematics in the Design of Clay Vessels:** Ethnomathematics plays a fundamental role in the design and creation of the clay vessels of the Pasto culture. Although at first glance it may seem like a purely artistic process, the design of these vessels involves a deep mathematical knowledge that is transmitted through tradition and experience.
- ii. **Geometry and Proportions:** Pasto artisans use geometric and proportional principles in the design of their vessels. For example, they use precise measurements to create symmetrical and balanced forms. The golden ratio, present in many works of art and architecture, can also be incorporated into the design of the vessels to achieve visual harmony.
- iii. **Patterns and Symbolism:** Geometric patterns and symbols carry deep cultural meaning in the clay vessels of the Pasto culture. These patterns may be based on simple geometric shapes such as circles, triangles, and squares, but their arrangement and combination have specific symbolism that reflects beliefs, stories, and traditions.
- iv. **Capacity and Functionality:** The design of the vessels is also related to their capacity and functionality. Pasto artisans use mathematical calculations to determine the size and shape of the vessels according to their purpose, whether for storing liquids, food, or other objects.
- v. **Technique and Manufacturing Process:** The process of making the vessels also involves mathematical skills. Artisans calculate the thickness of the clay, the firing temperature in the kiln, the drying time, and other technical aspects that require both mathematical knowledge and practical experience. (Ramírez, 2022)

i. **Relevance of Ethnomathematics in this Context:** The integration of ethnomathematics in the design of clay vessels in the Pasto culture has several significant implications:

- Cultural Preservation:

By understanding and valuing the mathematics involved in vessel design, the preservation of cultural traditions and ancestral knowledge passed down through generations is promoted.

- Appreciation of Mathematics:

This approach highlights the relevance and applicability of mathematics in everyday and cultural contexts, moving away from the perception of mathematics as abstract and disconnected from real life.

- Education and Learning:

Ethnomathematics in vessel design can be a powerful educational tool. It can be used to teach mathematical concepts such as geometry, proportions, patterns, and measurements in a tangible and meaningful way for students. (D'Ambrosio, 2013)

- Interdisciplinarity:

This approach fosters interdisciplinarity by integrating mathematics, art, history, and anthropology. Students can explore the connections between different areas of knowledge and understand how they intertwine in daily and cultural life.

ii. **Practical Application in Education:** In the educational field, ethnomathematics in the design of clay vessels from the Pasto culture can be a valuable tool for teaching mathematics in a contextualized and meaningful way. Some strategies that could be implemented include:

iii. **Research and Study:** Encourage students to research and study the design and manufacturing process of Pasto clay vessels, identifying the mathematical elements involved and their cultural significance.

iv. **Creation of Vessels:** Promote hands-on activities where students can create their own clay vessels using learned mathematical techniques such as geometry, proportions, and patterns.

v. **Analysis of Patterns and Symbolism:** Challenge students to analyze the geometric patterns and symbols present in the Pasto clay vessels, identifying their cultural meaning and their connection to mathematics.

vi. **Discussion and Reflection:** Facilitate discussions and reflections on the importance of ethnomathematics in cultural preservation, the appreciation of mathematics, and the connection between different areas of knowledge.

e. The Pasto Culture and the Polytechnic State University of Carchi (UPEC)

i. **Cultural Context of the Pasto Culture:** The Pasto culture is an ethnic group that inhabits the southwestern region of Colombia, especially in the department of Nariño and parts of Ecuador. They are known for their rich cultural heritage, which includes traditions, art, music, dance, and unique crafts. Among their most notable traditions are the celebrations of the Carnival of Blacks and Whites and the creation of crafts such as clay vessels, textiles, and traditional jewelry.

ii. **The Polytechnic State University of Carchi (UPEC):** The Polytechnic State University of Carchi is a higher education institution located in the province of Carchi, Ecuador. Its main mission is to provide quality education, promote scientific and technological research, and contribute to the sustainable development of the region and the country. UPEC is known for its focus on interculturality, inclusion, and intercultural dialogue.

iii. **Relationship and Collaboration:** The relationship between Pasto culture and the Polytechnic State University of Carchi has strengthened over time through various initiatives and collaborations:

iv. **Academic Programs:** UPEC has developed academic programs and courses that incorporate the history, culture, and knowledge of the Pasto community. These programs include anthropological, ethnographic, linguistic, and artistic studies that allow students and scholars to explore and understand the cultural richness of this community.

v. **Collaborative Research:** UPEC and Pasto community have carried out collaborative research projects in areas such as the preservation of cultural heritage, the study of ancestral traditions, the revitalization of the language, and the promotion of sustainable cultural tourism. These projects have generated both academic and practical knowledge that benefit both the university and the community.

vi. **Cultural Events and Celebrations:** UPEC has organized events, festivals, and cultural celebrations in collaboration with the Pasto community. These events include exhibitions of indigenous art, traditional music concerts, handicraft fairs, and intercultural gatherings that promote dialogue and the exchange of knowledge among different ethnic groups.

vii. **Human Resource Development:** UPEC has contributed to the training of professionals and community leaders from Pasto culture through training programs, study scholarships, internships, and university extension projects. This has strengthened local capacities and created opportunities for personal and professional development for community members.

viii. **Impact and Benefits:** The relationship between the Pasto culture and the Universidad Politécnica Estatal del Carchi (UPEC) has had a significant impact in various areas:

ix. **Cultural Preservation:** The collaboration has helped preserve and revitalize the cultural heritage of the Pasto community, including their traditions, language, art, and crafts. This is essential to ensure the continuity and recognition of the community's cultural identity.

x. **Educational Development:** The integration of Pasto culture into academic programs has enriched the education offered by UPEC, providing students with an intercultural perspective and a deeper understanding of the region's cultural diversity.

xi. **Knowledge Exchange:** The collaboration has facilitated the exchange of knowledge and experiences between the university and the community, promoting mutual learning, intercultural respect, and appreciation of traditional wisdom.

xii. **Promotion of Cultural Tourism:** The collaboration has contributed to the development of sustainable cultural tourism in the region, promoting visits from tourists interested in learning about the Pasto culture, its traditions, and its way of life.

f. Challenges and Future Opportunities

Despite the achievements made, there are still challenges and future opportunities in the relationship between the Pasto culture and the Universidad Politécnica Estatal del Carchi (UPEC):

i. **Sustainability:** It is important to ensure the long-term sustainability of collaborative initiatives and projects, guaranteeing continued benefits for both parties.

ii. **Innovation and Creativity:** New forms of collaboration can be explored to foster innovation, creativity, and the joint development of solutions for current and future challenges.

iii. **Inclusion and Participation:** It is essential to promote the inclusion and active participation of members of the Pasto community in all stages of the projects, ensuring their voices and perspectives are heard and valued.

iv. **Promotion of Diversity:** The relationship between UPEC and the Pasto community can serve as an example and model for promoting cultural diversity, intercultural dialogue, and peaceful coexistence in the region and beyond. (Leal, 2014)

g. Methodology

The methodology used for implementing ethnomathematics in the learning of Integral Calculus was the CDIO model, a specific approach developed by MIT for engineering courses. It is based on the idea that professionals must be able not only to design technical solutions but also to understand their impact on society, work in multidisciplinary teams, and adapt to changing environments. Therefore, this methodology aims to prepare students to face real-world challenges and be agents of change in their respective fields.

At the Universidad Politécnica Estatal del Carchi, the subject of Integral Calculus is taught in the engineering programs of logistics and transportation, computer science, agriculture, and food. This methodology allows students to acquire knowledge through a problem-solving process rooted in real professional challenges. The acronym CDIO includes the following stages:

i. **Conceive:** Students learn to identify and understand real-world problems and formulate innovative solutions to address them. This stage fosters creativity and critical thinking.

ii. **Design:** Students develop design skills to create practical and viable solutions for the problems identified during the conception stage. This involves applying theoretical knowledge to create products, systems, or processes.

iii. **Implement:** Students carry out the practical implementation of their designs using appropriate techniques and tools to execute their projects. This stage emphasizes applying classroom concepts to real-world settings.

i. **Operate:** Students learn to operate and maintain the products, processes, or systems they have developed, understanding how they function in real conditions and managing their performance over time. (Ibargüen Ocampo & Marín Hurtado, 2013)

In this study, the CDIO methodology stages are adapted to improve the learning of integral calculus based on the cultural interest in the Pasto community—specifically in calculating the surface area of traditional clay vessels—as follows:

i. **Conceive:** In this stage, students investigate the problem, which involves calculating the exact surface area of a vessel with a specific shape. They evaluate the available tools and data based on theory learned in class.

ii. **Design:** After obtaining the necessary data, students analytically develop the process to find the vessel's area and plan to use software to make the data collection more efficient.

iii. **Implement:** Implementation is done using Excel to derive the necessary equations, which are later used in GeoGebra to graph the 3D solid and calculate the area.

iv. **Operate:** Students must be capable of modifying parameters and maintaining the software so the model can adapt to different types of Pasto vessels, using basic functions to construct them.

Before delving into the details of integral calculus, it is important to understand that clay vessels have complex 3D shapes that can be modeled mathematically. For example, a vessel may have a cross-section resembling a function f(x) in the xy-plane. By rotating this function around an axis, a three-dimensional shape is created that represents the vessel.

To calculate the volume of a clay vessel using integral calculus and volumes of revolution, a general process is followed that involves integrating a specific function over a defined interval. Let us outline this process with a concrete example:

- The function f(x) represents the cross-sectional profile of the vessel. By rotating this function around the x-axis, a three-dimensional solid is generated that represents the full shape of the vessel.

$$V = \pi \int_a^b [f(x)]^2 dx$$

The vessel used in this research, characteristic of Pasto culture, has the shape of a truncated cone and features Pasto cultural designs on its exterior, as shown in Figure 1. In the first stage, students take measurements using measuring tape on parts they consider appropriate and useful for the next stage. In the course where ethnomathematics was applied, two different approaches were observed to calculate the surface area of the vessel, but both ultimately relied on the use of solids of revolution, a topic that had already been covered in class.

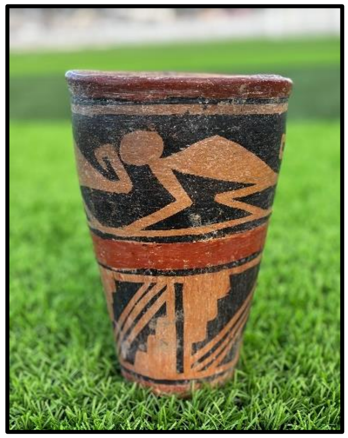

Figure 1. Vessel from the Pasto Culture

Figure 2. Measurements of the Vessel

The students chose as a solution to find a linear function that describes the slope of the vessel's lateral side, and by rotating it, they could determine the volume using the formulas previously learned in class. A significant group chose to measure the diameter at the base and at the top, which were 9 cm and 14 cm, respectively, along with the height of the vessel, which is 24 cm. Using these values, they constructed a Cartesian coordinate system to determine the function to be rotated, as shown in Figure 3.

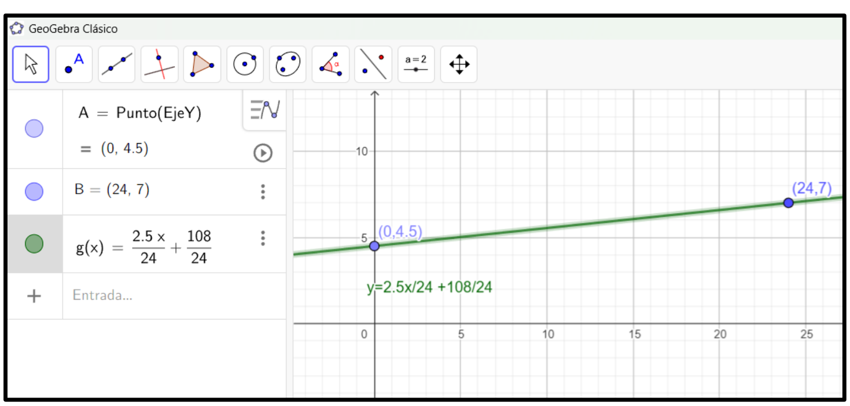

Figure 3. Linear Equation Representing the Lateral Slope of the Vessel

Other students chose to measure the diameter of the vessel every 2 cm in height to obtain the radius at each point where the vessel widens. With this data, they created a table in Excel, which, when graphed with a trendline, provided the equation that describes the lateral slope of the vessel.

Table 1: Radius as a Function of Height

Height (cm)	Radios (cm)
0	4,50
2	4,75
4	4,95
6	5,18
8	5,45
10	5,68
12	5,90
14	5,98
16	6,03
18	6,28
20	6,50
22	6,75
24	7,00

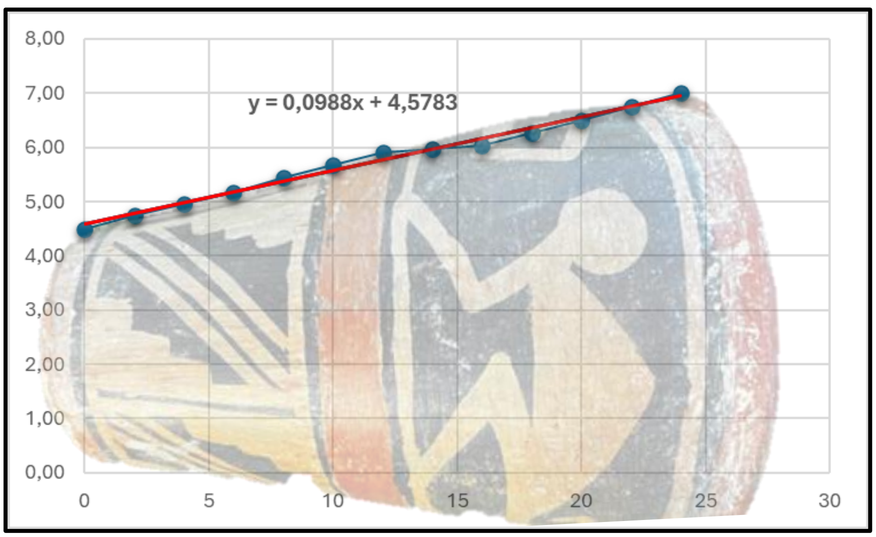

Figure 4. Equation of the Vessel's Lateral Slope

As can be observed, both methods used by the students result in the same equation, which accurately represents the lateral slope of the vessel. Using this equation in GeoGebra, the formula for the surface area of solids of revolution can be applied to calculate the surface area of the vessel, as shown in the following graph.

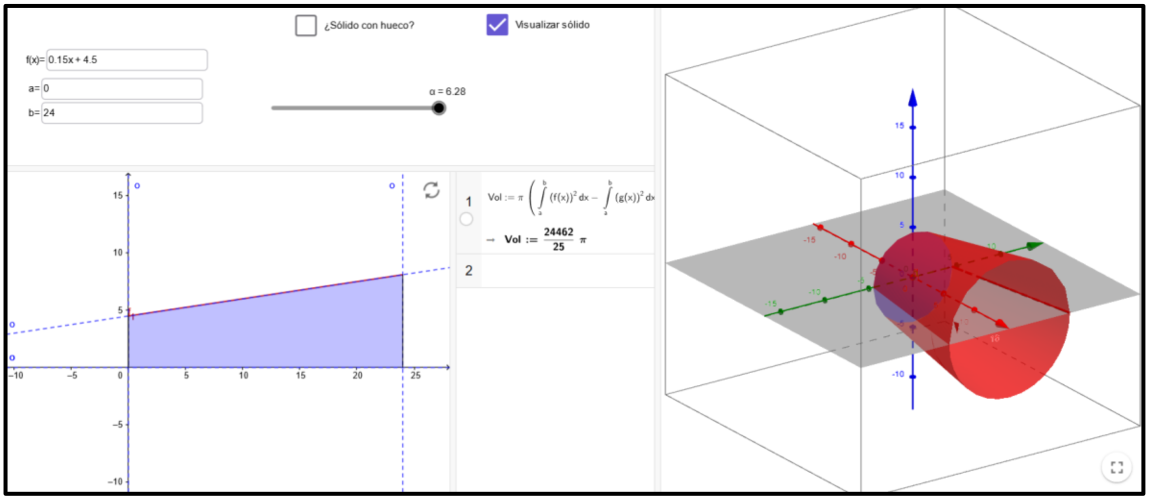

Figure 5. Rotation of the Linear Function

The result obtained in the GeoGebra program is $\frac{24462}{25}\pi$ The result obtained in the GeoGebra program is verified algebraically using the formulas for the surface area of solids of revolution.

Results

In the field of mathematics education, there are various pedagogical approaches aimed at improving students' understanding and real-life applicability of mathematical concepts. One of these approaches is the use of ethnomathematics, which seeks to integrate the mathematical practices of different cultures into the teaching and learning process. In this context, we will analyze a comparative example between two mathematics courses: one using a traditional approach (TC) and another using ethnomatical application (EC), focusing on the topic of volumes of revolution and the resulting differences in terms of understanding and applicability.

a. Traditional Course (TC):

In the traditional course, students are typically introduced to volumes of revolution in an abstract and decontextualized manner. Lessons focus on formal definitions, operational rules, and solving standard mathematical problems. While this method may provide a basic understanding of volumes of revolution, it often fails to connect this knowledge with students' everyday lives.

At the start of the course, students frequently struggle to grasp the concept of volumes of revolution, as they are presented as strange, abstract numbers with no clear connection to their environment or daily experiences. Throughout the semester, theoretical classes are held to explain the properties of volumes of revolution, basic operations (addition, subtraction, multiplication, division) are taught, and practice exercises are solved. However, most of these exercises remain purely mathematical, lacking connection to real-world situations that students may encounter.

As the course progresses, some students improve their ability to solve problems involving volumes of revolution, but they still face difficulties in applying this knowledge beyond the academic context. The lack of real-world relevance limits their deep understanding and the practical usefulness of the concept.

i. Course with Ethnomathematical Application (EC):

In contrast, the course that integrates ethnomathematics aims to connect the mathematical practices of different cultures with the teaching and learning of volumes of revolution. From the beginning, students are encouraged to explore how volumes of revolution are used in their own cultures and surroundings. This approach allows students to find meaning and relevance in studying volumes of revolution from a broader, more diverse perspective.

Throughout the semester, activities and examples from different cultures are incorporated, showing practical and contextualized uses of volumes of revolution. Students not only learn the properties and operations involved but also explore how these are used in everyday situations such as cooking, handicrafts, time measurement, and architectural design, among others.

A core element of the ethnomathematical approach is to encourage discussion and idea exchange about how different cultures use volumes of revolution in daily life. This not only enriches students' learning by offering multiple perspectives but also helps them relate the mathematical concept to their cultural and social context.

b. Comparative Results:

By the time of the final exam, significant differences can be observed between the two courses:

i. Deep Understanding:

Students in the TC course show a basic understanding of volumes of revolution. They can perform calculations and solve standard problems but lack practical experiences to understand the true utility and meaning of the concept.

On the other hand, EC course students demonstrate a deeper understanding. They can connect the concept to their daily lives and the mathematical practices of different cultures, allowing them not only to solve problems but also to apply the concept creatively and contextually in diverse situations.

ii. Real-World Applicability:

Students in the TC course tend to apply their knowledge only in academic or abstract scenarios and find it difficult to transfer this knowledge to real-life contexts.

In contrast, EC course students develop a stronger ability to apply volumes of revolution in practical and relevant situations. They explore how these concepts are used in different cultures and how those applications have direct impact on their environment and daily activities.

iii. Motivation and Engagement:

The ethnomathematical approach fosters higher levels of motivation and participation among students. By connecting volumes of revolution to their own cultural experiences, students feel more engaged and enthusiastic about learning.

In contrast, some students in the traditional course may lose interest or disengage due to the lack of connection to their personal reality and experiences.

Discussion

The use of ethnomathematics in the design of clay vessels from the Pasto culture illustrates how mathematics is inherently connected to creativity, culture, and the history of a community. This approach highlights the importance of recognizing and valuing mathematical practices within specific cultural contexts, promoting a more inclusive, relevant, and meaningful mathematics education for students. Integrating ethnomathematics into the educational setting can enrich math teaching by showing its real-life applicability and its interconnection with other disciplines and cultural traditions.

The application of ethnomathematics in teaching volumes of revolution has proven to generate more meaningful results in terms of deep understanding and real-world applicability compared to the traditional approach. By incorporating mathematical practices from different cultures, student learning is enhanced, a broader and more diverse view of mathematics is promoted, and greater motivation and participation in the educational process is fostered. This example illustrates how pedagogical approaches can have a significant impact on student outcomes and learning experiences—not only in mathematics but across disciplines.

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